Ingeniería e Innovación
RIINN
Revista Científica Facultad de Ingeniería

Revista Ingeniería e Innovación Volumen 13 Número 1, Enero-Junio 2025 e-ISSN:2346-0474 (en línea)

Modeling stock market crash dynamics using analysis of anti-plane shear deformation of a cracked finite isotropic wedge.

Modelado de la dinámica de caídas de la bolsa de valores mediante el análisis de la deformación por corte antiplano de una cuña isotrópica finita agrietada.

Silas Abahia Ihedioha
Department of Mathematics, Plateau State University

Bright Okore Osu
Department of Mathematics, Abia State University
https://orcid.org/0000-0003-2463-430X

Chidinma Olunkwa
Department of Mathematics, Abia State University
https://orcid.org/0000-0003-4916-0753

Carlos Granados
Escuela de Ciencias de la Educación, Universidad Nacional Abierta y a Distancia
https://orcid.org/00000-0002-7754-1468

carlos Granados, Abahia Ihedioha, S., Okore Osu, B. ., & Olunkwa, C. . (2025). MODELING STOCK MARKET CRASH DYNAMICS USING ANALYSIS OF ANTI-PLANE SHEAR DEFORMATION OF A CRACKED FINITE ISOTROPIC WEDGE. Ingeniería E Innovación, 13(1). https://doi.org/10.21897/rii.3862

Recibido:30/03/2025 *Aprobado:10/04/2025* *Publicado:19/05/2025*

ABSTRACT

This paper introduces a novel approach to modeling stock market crash dynamics by drawing an analogy to the mechanics of anti-plane shear deformation in a cracked finite isotropic wedge. Financial markets under stress exhibit patterns of stress concentration, failure propagation, and systemic breakdown, much like cracked elastic media under mechanical deformation.

The main objective of this study is to develop a mathematical model that describes the evolution of financial crises as mechanical fracture processes in elastic media. To this end, the specific objectives are: (1) to establish a formal correspondence between the physical variables of the elastic model and financial indicators of risk and instability; (2) to derive critical conditions for crisis propagation using methods from elasticity theory and fracture mechanics; and (3) to analyze the influence of parameters such as the shear modulus (μ), interpreted as market resilience, on the dynamics of recovery or collapse. This will be achieved by using methods from fracture mechanics and elasticity theory, we derive a framework that captures the spread of financial distress, the collapse of price stability, and the conditions for market recovery. The model provides insights into liquidity crises, systemic risk, and intervention strategies.

Keywords: Stock Market Crash, Anti-plane Shear Deformation, Fracture Mechanics, Systemic Risk, Liquidity Crisis, Financial Contagion, Market Stability

RESUMEN

Este artículo presenta un enfoque novedoso para modelar la dinámica de las caídas bursátiles mediante una analogía con la mecánica de la deformación por cizallamiento antiplanar en una cuña isótropa finita agrietada. Los mercados financieros sometidos a tensión presentan patrones de concentración de tensiones, propagación de fallos y colapso sistémico, similares a los de los medios elásticos agrietados sometidos a deformación mecánica.

El objetivo principal de este estudio es desarrollar un modelo matemático que describa la evolución de las crisis financieras como procesos de fractura mecánica en medios elásticos. Para ello, se plantean como objetivos específicos: (1) establecer una correspondencia formal entre las variables físicas del modelo elástico y los indicadores financieros de riesgo e inestabilidad; (2) derivar condiciones críticas para la propagación de crisis mediante métodos de la teoría de elasticidad y mecánica de fracturas; y (3) analizar la influencia de parámetros como el módulo de corte (μ), interpretado como la resiliencia del mercado, en la dinámica de recuperación o colapso. Esto se logrará, utilizando métodos de la mecánica de fracturas y la teoría de la elasticidad, derivamos un marco que captura la propagación de las dificultades financieras, el colapso de la estabilidad de precios y las condiciones para la recuperación del mercado. El modelo proporciona información sobre las crisis de liquidez, el riesgo sistémico y las estrategias de intervención.

Palabras claves: Caída de la bolsa, deformación por cizallamiento antiplanar, mecánica de fracturas, riesgo sistémico, crisis de liquidez, contagio financiero, estabilidad del Mercado.

INTRODUCTION

A stock market crash is a sudden, sharp decline in stock prices across major indices, often exceeding 20% over a short period, leading to economic instability (Mishkin, 1991). These crashes typically result from speculative bubbles, macroeconomic shocks, or systemic financial imbalances (Kindleberger & Aliber, 2011).

Stock market crashes are characterized by several key features. First, they involve a rapid and severe decline, where stock prices plummet within days or weeks. A historical example is the 1929 crash when the Dow Jones Industrial Average (DJIA) lost 89% over three years (Galbraith, 1954). Second, there is high market volatility, with extreme price fluctuations often measured by the VIX, which spiked during the 1987 Black Monday crash (Whaley, 2009). Third, panic selling occurs as fear-driven mass selloffs lead to liquidity shortages, exacerbating the crisis (Kindleberger & Aliber, 2011). Fourth, systemic contagion spreads the crash beyond the stock market, affecting bond, foreign exchange, and commodity markets, amplifying global economic risks (Brunnermeier & Pedersen, 2009). Finally, crashes have economic consequences, often triggering recessions, bank failures, and unemployment, as seen in the 2008 Global Financial Crisis (Reinhart & Rogoff, 2009).

Several stock market crashes have had profound economic impacts. The 1929 Great Crash, triggered by speculation and margin trading, led to the Great Depression (Galbraith, 1954). The 1987 Black Monday saw a 22.6% single-day drop in the DJIA due to program trading and illiquidity (Roll, 1988). The 2000 Dot-Com Bubble involved the collapse of overvalued tech stocks, erasing trillions in market value (Shiller, 2005). The 2008 Global Financial Crisis was driven by subprime mortgage failures, causing a 50% stock market decline (Reinhart & Rogoff, 2009). More recently, the 2020 COVID-19 Crash saw a pandemic-induced panic leading to the fastest-ever 30% drop in the S&P 500 (Baker et al., 2020).

Crashes often stem from speculative bubbles (Minsky, 1986), excessive leverage (Adrian & Shin, 2010), and external shocks such as wars or pandemics (Barro, 2006). High-frequency trading has also exacerbated market disruptions, as observed in the 2010 Flash Crash (Kirilenko et al., 2017; Ihedioha et al., 2024).

Governments and central banks intervene using circuit breakers, monetary easing, and fiscal stimulus to stabilize markets (Bernanke, 2015; Ador et al., 2024). For example, after the 1987 crash, regulatory measures such as trading halts were implemented to prevent future panics (Roll, 1988). While regulatory responses can mitigate the impact of crashes, market instability remains an inherent risk in financial systems (Kindleberger & Aliber, 2011).

A stock market crash can be likened to the sudden propagation of financial stress, similar to how

cracks develop in solid materials under stress. The Analysis of Antiplane Shear Deformation of a Cracked Finite Isotropic Wedge provides a powerful analogy for modeling crash dynamics by drawing parallels between financial system stability and fracture mechanics principles.
In fracture mechanics, a material under stress accumulates energy until it reaches a critical point, after which a crack propagates, leading to failure (Anderson, 2017; Ihedioha et al., 2024). Similarly, in financial markets, economic pressures (e.g., excessive leverage, speculative bubbles) build up over time. Once stress reaches a threshold, market fractures (crashes) occur, releasing stored instability in a nonlinear, cascading manner.

Anti-plane shear deformation considers out-of-plane displacement in a cracked solid subjected to shear forces (Sih, 1973). This concept can be mapped onto financial stress propagation during a crash:

Market Participants as Material Elements: Traders, institutions, and investors function like atomic structures in a material, transmitting stress just as atoms transfer mechanical loads.
Crack Tip as Systemic Risk Concentration: In financial markets, the “crack tip” represents a highly leveraged institution or a vulnerable asset class (e.g., subprime mortgages in 2008). When stress exceeds a critical level, the crack (financial collapse) expands dynamically.
Stress Intensity Factor (SIF) and Crash Magnitude: The stress intensity factor (K-parameter) in fracture mechanics determines whether a crack propagates. In stock markets, the equivalent parameter could be liquidity stress and margin call pressure, dictating whether a crash accelerates or stabilizes.
By applying fracture mechanics principles to financial markets, it becomes possible to better understand the nonlinear dynamics of stock market crashes and systemic financial instability.
Mathematical Modeling: Crack Propagation in a Finite Wedge and Market Instability

The governing equation for anti-plane shear deformation in an isotropic wedge is:

$$\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$$

In this case, we start from the constitutive law of the shear stress for an anti-plane shear problem:

$$\tau_{xz} = \mu \frac{\partial \tau_{xz}}{\partial x}$$

And

$$\tau_{yz} = \mu \frac{\partial \tau_{yz}}{\partial y}$$

Where

$$\mu \frac{\partial \tau_{xz}}{\partial x} + \mu \frac{\partial \tau_{yz}}{\partial y} = 0$$

nd this should be replace in the first equation, where:

w(x,y) represents out-of-plane displacement (analogous to financial market displacements).

μ is the shear modulus (analogous to market liquidity resilience).

∇^2represents the Laplacian, governing stress diffusion.

In a **cracked wedge geometry**, the stress field near the crack tip follows the asymptotic form:

$$\sigma_{\theta z} \sim K_{\mathrm{III}} r^{\frac{-1}{2}} f\left(\theta, \alpha\right)$$

where:

- σ_θzis the shear stress component.
- *K_III* is the **Mode III Stress Intensity Factor**, dictating crack growth (analogous to systemic risk amplification).
- *r^((-1)/2)* shows stress singularity at the crack tip (analogous to crisis escalation at breaking points).
- *f(θ,α)*describes how stress distributes around the wedge (analogous to different market sectors absorbing shocks).

This formulation describes how m**arket stress localizes and then propagates in a nonlinear manner**, similar to crack growth in fracture mechanics.

In the context of fracture mechanics, stock market crashes can be interpreted through the wedge model, where financial instability mimics the accumulation and propagation of stress in materials. Just as stress accumulates at a pre-existing flaw in a material, financial systems build up risk concentrations due to factors like overleveraging and asset bubbles. This phase represents the gradual accumulation of systemic vulnerabilities before a crash. When systemic risk surpasses a critical threshold *(K_III > K_c)*, an unstable rupture occurs, leading to a financial crash that propagates through interconnected markets. The crack angle *(α)* determines the direction of crash propagation, akin to how a financial collapse can spread to derivatives, credit markets, or international stock exchanges. External interventions, such as monetary policies and fiscal measures, act as “fracture toughening mechanisms” that increaseK_c *(market resilience)* and halt further collapse. For instance, quantitative easing *(QE)* functions as a form of fracture energy dissipation, slowing down systemic contagion in the same way that crack-tip plasticity absorbs mechanical energy in materials.

The wedge model analogy suggests that financial regulators can enhance market stability by adopting strategies similar to those used in material stress management. Monitoring stress intensity factors (K-parameters) such as volatility, leverage ratios, and liquidity gaps allows for the early detection of systemic vulnerabilities. Identifying critical “crack tips,” or fragile financial sectors, can help prevent widespread instability by addressing localized stress concentrations before they trigger a larger collapse. Developing shock-absorbing mechanisms or stress redistribution strategies can slow market contagion, much like engineers use toughened materials to prevent catastrophic fractures. By applying anti-plane shear deformation analysis of a cracked finite isotropic wedge to stock market crash dynamics, a structured mathematical framework emerges for modeling systemic risk propagation. This approach provides deeper insights into how financial stress localizes and spreads, enabling the development of more effective market stabilization strategies to prevent severe crashes.

Anti-plane shear deformation is a valuable mathematical framework for modeling financial stress concentration and systemic risk propagation during stock market crashes. The justification for its use in financial modeling is based on several key analogies. In fracture mechanics, anti-plane shear deformation describes the out-of-plane displacement of a cracked solid under shear forces. Similarly, in financial markets, market stress propagates through interconnected institutions just as shear stress spreads through a solid material. Liquidity shortages and leverage imbalances create localized stress concentrations, analogous to stress singularities at a crack tip.

Anti-plane shear deformation captures stress intensification near crack tips, mirroring how financial instability escalates. The Mode III Stress Intensity Factor *(K_III)* governs crack growth, just as liquidity stress intensity dictates market contagion. When *K_III* exceeds a critical threshold, a crack propagates—similarly, a financial collapse spreads once systemic risk surpasses a tipping point. Stock market crashes exhibit nonlinear behavior, where small disturbances can trigger large-scale collapses. Anti-plane shear models account for singular stress fields, representing instability concentration at market tipping points, and fracture propagation direction (α), reflecting how risk spreads across different asset classes and financial institutions.

A finite isotropic wedge serves as a model for financial systems where stress is distributed unevenly across sectors. Sharp wedge angles represent highly concentrated markets, such as overleveraged hedge funds, whereas blunted wedges correspond to diversified systems with lower systemic risk. Using anti-plane shear deformation provides a mathematically rigorous and physically intuitive approach to studying financial stress propagation. It allows for the identification of systemic risk concentrations, the prediction of market contagion paths, and the design of stabilization mechanisms to prevent severe crashes.

1. LITERATURE REVIEW

Stock market crashes are rapid, large-scale declines in asset prices that often lead to systemic financial crises. Theories explaining crashes range from speculative bubbles and leverage dynamics to liquidity shocks and network contagion.
One foundational framework for understanding financial crashes is Minsky's Financial Instability Hypothesis (Minsky, 1986), which suggests that financial markets naturally evolve from stability to instability as risk-taking increases. Kindleberger & Aliber (2011) expanded on this idea, emphasizing that speculative bubbles form when investor euphoria leads to asset overvaluation, followed by a sharp correction.

Another important perspective comes from behavioral finance, where irrational investor behavior exacerbates volatility. Shiller (2005) introduced the concept of irrational exuberance, showing how herd behavior and overconfidence can inflate bubbles. When sentiment shifts, panic selling drives market collapses.
Network-based theories focus on financial contagion, where distress spreads through interconnected institutions (Acemoglu et al., 2015). Brunnermeier & Pedersen (2009) linked liquidity constraints and funding liquidity risk to crash dynamics, showing how declining collateral values trigger forced liquidations and systemic collapse.

1.1. . Empirical Studies on Major Market Crashes

Historical market crashes provide critical insights into crash dynamics and their aftermath:

1929 Great Depression – Fueled by speculation and margin trading, the stock market lost 89% of its value, leading to global economic contraction (Galbraith, 1954).
1987 Black Monday – The DJIA fell by 22.6% in a single day, attributed to program trading and illiquidity (Roll, 1988).
2000 Dot-Com Bubble – Overvaluation of tech stocks led to a 78% NASDAQ decline, highlighting the dangers of speculative investment (Shiller, 2005).
2008 Global Financial Crisis – Excessive mortgage-backed securities leverage caused systemic collapse, demonstrating the role of financial innovation in market crashes (Reinhart & Rogoff, 2009).
2020 COVID-19 Crash – A rapid 34% decline in the S&P 500 due to pandemic uncertainty underscored the role of external shocks in market instability (Baker et al., 2020).

These studies reinforce that crashes exhibit nonlinear, fractal-like behavior, making them analogous to mechanical fracture propagation.

Fracture mechanics, originally developed to study material failure under stress, provides an analogical and mathematical framework for financial instability. The financial system can be seen as an interconnected structure where stress accumulates and propagates, much like mechanical fractures.

Fracture mechanics examines how stress concentration near cracks leads to failure. According to Griffith's energy criterion (Griffith, 1921), crack growth occurs when the strain energy release rate exceeds the material's fracture toughness. Paris & Erdogan (1963) later introduced the Paris law, describing crack growth rate under cyclic loading.

Cracks propagate in different modes (Anderson, 2017):
Mode I (Opening Mode) – Analogous to credit defaults causing liquidity gaps.
Mode II (Sliding Mode) – Similar to arbitrage breakdown leading to pricing inefficiencies.
Mode III (Anti-plane Shear Mode) – Relevant to systemic risk propagation in financial networks.

Mathematical parallels between fracture propagation and financial contagion have been explored in recent literature. Peters & Klein (2013) used percolation theory to model market stress diffusion, showing that financial crashes resemble cascading mechanical failures.
Gabaix et al. (2003) applied power-law scaling from fracture mechanics to stock market movements, arguing that market crashes exhibit self-organized criticality similar to material failure. Filimonov & Sornette (2012) extended these ideas, using crack propagation models to simulate stock price collapses.
These studies suggest that stress intensity factors (SIFs) in fracture mechanics correspond to market risk intensity factors, providing a structured way to model crash dynamics.

In mechanics, anti-plane shear deformation describes out-of-plane displacement in a cracked medium under shear loading. It is governed by the Laplace equation (Sih, 1973):

$$\mu\nabla^2 w = 0$$

where

w(x,y) is the displacement field, and μ is the shear modulus. The Mode III stress intensity factor *(K_III)* dictates the crack's growth rate:

$$\sigma_{\theta z} \sim K_{\mathrm{III}} r^{\frac{-1}{2}} f\left(\theta, \alpha\right)$$

where

σ_θz is the shear stress component, and $r^{\frac{-1}{2}}$ signifies a singularity at the crack tip.
This model explains how localized stress propagates and leads to system-wide failure, making it highly relevant for modeling financial contagion. A Financial Interpretation of Anti-plane Shear Deformation can be seen as follows:

- Applying anti-plane shear models to financial markets provides insights into systemic risk dynamics: Shear stress as financial instability: Just as stress concentrates at crack tips, financial stress builds up at key institutions (e.g., Lehman Brothers in 2008).

- Crack propagation as crisis contagion: When market instability exceeds a threshold
- *(K_III>K_c)*, risk spreads nonlinearly across financial sectors.
- Wedge geometry as market segmentation: A finite isotropic wedge represents how different asset classes (e.g., equities, bonds, derivatives) absorb and transfer financial stress.

Recent studies have applied elasticity theory to financial stress modeling. Balanki n & Susarrey (2005) used fracture mechanics analogies to describe stock price movements, while Dacorogna et al. (2001) found that financial time series exhibit stress-strain characteristics similar to materials under load.

Empirical studies support the analogy between fracture propagation and financial crises. Preis et al. (2011) showed that stock price fluctuations follow power-law distributions, consistent with crack growth models. Mandelbrot (1997) found that financial time series exhibit multifractal scaling, mirroring stress fields in fractured materials.
The integration of fracture mechanics and anti-plane shear deformation into financial modeling provides a quantitative framework for understanding market crashes and systemic risk propagation. Financial crises exhibit nonlinear, stress-concentrated behavior, making Mode III shear deformation particularly relevant for modeling contagion dynamics.
Future research should explore how stress intensity factors (K-parameters) in financial networks correspond to mechanical fracture criteria, allowing for more robust predictive models of market instability.

2. MATHEMATICAL FORMULATION OF ANTIPLANE SHEAR DEFORMATION AND ITS FINANCIAL INTERPRETATION

Governing Equation: The Laplace Equation in Anti-plane Shear Deformation
In elasticity theory, anti-plane shear deformation describes a situation where the displacement occurs perpendicular to the plane of interest, and the stress distribution follows a Laplace equation.

Mathematically, the governing equation is:

$$\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0, \tag{1}$$

where:
w(x,y) is the out-of-plane displacement field, which in this context represents the financial stress function over spatial coordinates *(x,y)*.

$\nabla^2 w$ is the Laplacian operator, describing how stress diffuses in the market.
This equation is harmonic, implying that financial stress redistributes itself smoothly unless disturbed by external forces (such as shocks or cracks).
Financial Interpretation of the Displacement Field w*(x,y)*
In finance, we draw an analogy:
w(x,y) represents financial stress, similar to how elastic displacement describes material deformation.
Regions with high ∇^2 wcorrespond to areas of concentrated financial pressure, such as markets experiencing liquidity crises, extreme volatility, or investor panic.
Steady-state solutions $\left(\nabla^2 w = 0\right)$ correspond to stable financial markets, where no abrupt disruptions occur.
A disturbance to this equilibrium—such as a market shock or the failure of a financial institution—can act as a source of instability, much like a crack in an elastic medium.

Boundary Conditions: Representation of External Shocks

To model financial stress propagation, we impose boundary conditions that represent real-world market scenarios.

(a) ***Dirichlet Boundary Conditions (Fixed Stress Level at Boundaries)***

$$w(x, y) = w_0 \text{on} \mathit{\Gamma}_1. \quad (2)$$

This models external interventions (e.g., central bank actions or regulations) that impose a fixed level of financial stability at certain boundaries.

(b) ***Neumann Boundary Conditions (External Shock-Induced Stress Gradient)***

$$\frac{\partial w}{\partial n} = S\left(x, y\right) \text{on} \mathit{\Gamma}_2, \quad (3)$$

where *S(x,y)* is a stress function representing external shocks, such as:
Liquidity Crises: Large withdrawals of funds, triggering instability.
Investor Panic: Sudden market sell-offs that amplify stress.
Systemic Risk Events: Bank failures, credit crunches, or geopolitical risks.
These conditions define how financial stress evolves at the edges of the system, mimicking external interventions and constraints.

Crack Representation: Modeling Market Weaknesses

A crack in an elastic medium corresponds to financial vulnerabilities such as:
Highly leveraged institutions, which amplify risk.
Illiquid assets, which restrict market adaptability.
Systemic interconnections, where stress propagates between institutions.
(a) Crack Modeled as a Stress-Free Surface

A crack in an anti-plane shear model satisfies the zero-traction condition on its surface:

$$\frac{\partial w}{\partial n} = 0, \tag{4}$$

on the crack faces.
This implies that once a financial institution (or market sector) collapses, it no longer bears financial stress but instead acts as a pathway for stress propagation to the rest of the system.
(b) Stress Intensity Factor (SIF): Risk Concentration around the Crack Tip
Near the crack tip, the stress field behaves as:

$$w(r, \theta) \approx K\sqrt{r} f(\theta), \tag{5}$$

where K is the Stress Intensity Factor (SIF), which determines how much financial pressure concentrates at critical points.
If K exceeds a critical threshold *(K_c)*, systemic failure occurs, leading to market collapse.
If K remains below, the system absorbs stress without catastrophic failure.
This allows us to quantify financial fragility and predict whether a crisis will spread or stabilize.

Financial Implications of This Model

Market Fragility and Contagion Risk:

- Highly leveraged institutions act as pre-existing cracks.
- Stress concentrates at weak points (e.g., hedge funds, banks with high exposure to risky\ assets).
- If financial pressure exceeds a threshold, the system undergoes systemic failure, analogous to crack propagation in materials.

Crash Dynamics and Liquidity Waves:

- External shocks (e.g., regulatory changes, economic downturns) modify boundary conditions, shifting stress distribution.
- Liquidity crises appear as stress singularities, where stress intensity grows beyond market resilience.
- Crack growth models can predict financial collapses and recovery paths.

Policy and Regulation:

- Targeting high-stress regions (analogous to crack tips) prevents further damage.
- Controlling stress intensity (via interest rates, liquidity injections, and bailouts) stabilizes financial systems.

This model provides a rigorous fracture mechanics-based framework for stock market crashes: The Laplace equation models stress redistribution in financial systems where boundary conditions capture real-world financial shocks (liquidity crises, regulatory changes, investor behavior) and cracks represent vulnerabilities in market structures, allowing for predictive modeling of systemic risks.

3. THE SOLUTION OF THE MODEL WITH DISCUSSIONS

We will now construct complete solutions to the Laplace equation:

$$\nabla^2 w = \frac{\partial^2}{\partial^2} w/\partial^2 x^2 + \frac{\partial^2}{\partial^2} w/\partial^2 y^2 = 0,$$

in a finite wedge domain with a crack along the radial direction and subject to various external financial stress conditions. We consider different boundary conditions separately and solve accordingly.
Convert Laplace's Equation to Polar Coordinates
Since the domain is a wedge-shaped region with a crack along the radial direction, it is convenient to use polar coordinates *(r, θ),* where:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \tag{6}$$

In polar coordinates, the Laplace equation transforms into:

$$\frac{\partial^2}{\partial r^2} w + \frac{1}{r}\frac{\partial}{\partial r} w + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} w = 0. \tag{7}$$

Separation of Variables

We assume a separable solution:

$$w(r,\theta) = R(\tilde{r})\Theta(\theta). \tag{8}$$

Substituting into the Laplace equation:

$$R''\Theta + \tfrac{1}{r}R'\Theta + \tfrac{1}{r^2}R\Theta'' = 0. \tag{9}$$

Dividing equation (9) by $R\Theta$ we obtain

$$\frac{R''}{R} + \frac{1R'}{rR} + \frac{1\Theta''}{r^2\Theta} = 0 \tag{10}$$

Multiplying equation (10) by r^2 and rearranging gives,

$$r^2\frac{R''}{R} + r\frac{R'}{R} = \frac{\Theta''}{\Theta} = \lambda, \tag{11}$$

where λ is the separation constant.

This gives two separate ordinary differential equations:

1. Radial equation:

$$r^2R'' + rR' - \lambda R = 0. \tag{12}$$

2. Angular equation:

$$\Theta'' + \lambda\Theta = 0. \tag{13}$$

We now solve these for different boundary conditions.
Solve the Angular Equation
The angular equation is:

$$\Theta'' + \lambda\Theta = 0.$$

To obtain the general solution we assume an exponential solution of the form

$$\Theta\left(\theta\right) = e^{m\theta}. \tag{14}$$

from which we get

$$\begin{Bmatrix} \Theta' = me^{m\theta} \\ \Theta'' = m^2e^{m\theta} \end{Bmatrix} \tag{15}$$

Substituting equationn (15) into equation (13), we obtain,

$$m^2e^{m\theta} + \lambda e^{m\theta} = 0. \tag{16}$$

Since e^mθ ≠0, we get the characteristic equation:

$$m^2 + \lambda = 0 \tag{17}$$

form which we get

$$m = \pm i\sqrt{\lambda} \tag{18}$$

Thus, the general solution for Θ(θ) is given by,

$$\theta\left(\theta\right) = C_1\cos\left(\sqrt{\lambda}\theta\right) + C_2\sin\left(\sqrt{\lambda}\theta\right) \tag{19}$$

This represents the oscillatory behavior, typical for angular equations in cylindrical or spherical coordinates.
Angular Boundary Conditions
We impose different boundary conditions based on the problem setup.

Stress-Free Crack (Symmetric Condition)
If the crack is stress-free, we require no shear stress at the crack edges $\theta = 0$ and $\theta = \alpha$:

$$\frac{\partial w}{\partial\theta} = 0 \text{ at } \theta = 0, \alpha \tag{20}$$

This implies that from equation (18),

$$\sin\left(\sqrt{\lambda}\theta\right) = 0 \text{ for } \theta = \alpha \tag{21}$$

Thus, √λ must take discrete values,

$$\sqrt{\lambda} = \frac{n\pi}{\alpha}, n = 0, 1, 2, \ldots \tag{22}$$

So the Eigen functions are,

$$\theta_n\left(\theta\right) = C_n \cos\left(\frac{n\pi}{\alpha}\theta\right) \tag{23}$$

where n is an integer.
Solve the Radial Equation
The radial equation as shown in equation (12) is;

$$r^2 R'' + rR' - \lambda R = 0 \tag{24}$$

Using the power-law assumption given as;

$$R(r) = r^m, \tag{25}$$

from which we get

$$\left.\begin{aligned} R' &= mr^{m-1} \\ R'' &= m\left(m-1\right)r^{m-2} \end{aligned}\right\}. \tag{26}$$

Substituting equation (26) into equation (24) yields:

$$r^2\left[m\left(m-1\right)r^{m-2}\right] + r\left[mr^{m-1}\right] - \lambda r^m = 0. \tag{27}$$

From the expansion of equation (27), we obtain;

$$m(m-1)r^m + mr^m - \lambda r^m = 0, \tag{28}$$

Which simplifies to

$$r^m[m(m-1) + m - \lambda] = 0. \tag{29}$$

after factoring r^m in equation (28).
Since $r^m \neq 0$, the characteristic equation is:

$$m(m-1) + m - \lambda = 0, \tag{30}$$

which simplify:

$$m^2 - \lambda = 0, \tag{31}$$

and

$$m = \pm\sqrt{\lambda}. \tag{32}$$

Thus, the general solution for R(r), equation (25), is:

$$R\left(r\right) = Ar^{\sqrt{\lambda}} + Br^{-\sqrt{\lambda}}, \tag{33}$$

where A and B are arbitrary constants.
From the eigenvalues found earlier,

$$\lambda = \left(\frac{n\pi}{\alpha}\right)^2,\ \text{so:} m_n = \pm\frac{n\pi}{\alpha}. \tag{34}$$

Thus, the general radial solution is

$$R_n(r) = A_n r^{\frac{n\pi}{\alpha}} + B_n r^{-\frac{n\pi}{\alpha}}. \tag{35}$$

Construct the Full Solution

Since the full solution is a sum over all modes:

$$w(r,\theta) = \sum_{n=0} \left(A_n r^{\frac{n\pi}{\alpha}} + B_n r^{-\frac{n\pi}{\alpha}}\right)\cos\left(\frac{n\pi}{\alpha}\theta\right) \tag{36}$$

where A_n and B_n are constants determined by boundary conditions.

Boundary Conditions

To obtain a final solution to the Laplace equation in a finite wedge domain with a radial crack, we need to choose appropriate boundary conditions and then graph the solution.

We choose boundary conditions as follows

Crack Boundary Condition (Zero Shear Stress)

The crack along $\theta = 0$ and $\theta = \alpha$ is stress-free:

This implies a Neumann condition:

$\partial w/\partial\theta=0$ for $\theta = 0,\alpha$.

This ensures symmetry, leading to cosine solutions.

Fixed Financial Stress at Outer Boundary $r = R$

We impose a Dirichlet boundary condition:

$w(R,\theta) = g(\theta)$, where $g(\theta)$ represents an imposed market stress function.

Regularity Condition at the Crack Tip, $r = 0$

We avoid singularities by requiring a bounded solution at $r = 0$.

This implies we keep only positive power terms of r.

We consider different Outer Boundary Conditions like;

Dirichlet Condition at r = R

If a fixed financial stress;

$$w(R,\theta) = g(\theta) \tag{37}$$

is imposed at r = R, we expand g(θ) in a Fourier series as:

$$g(\theta) = \sum_{n=0} C_n \cos\left(\frac{n\pi}{\alpha}\theta\right). \tag{38}$$

Matching terms in equations (36 and (37))at r = R gives;

$$A_n R^{\frac{n\pi}{\alpha}} + B_n R^{-\frac{n\pi}{\alpha}} = C_n. \tag{39}$$

Solving for A_n,B_n, we now obtain the expected result.

Neumann Condition at r = R

If the financial stress flux vanishes:

$$\frac{\partial w}{\partial r} \mid r = R = 0. \tag{40}$$

The differentiation of equation (36) gives;

$$\sum_{n=0}^{\infty}\left(An\frac{n}{\pi}^{\frac{n}{\pi}}R^{\frac{n\pi}{\alpha}} - Bn\frac{n}{\pi}^{\frac{n\pi}{\alpha}-1}\right)\cdot\cos\left(\frac{n\pi}{\alpha}\theta\right) = 0, \tag{41}$$

which leads to a system for A_n,B_n.

Stress Decay at Infinity *(r → ∞)*

If the financial stress must vanish at large distances, we require:

$$\boldsymbol{w(r,\theta)\to 0 \quad as\ r\to\infty.} \tag{42}$$

This implies that it is only the decaying term that remains and;

$$\boldsymbol{Bn = 0.} \tag{43}$$

Thus,

$$w(r,\theta) = \sum_{n=0} A^n r^{\frac{n\pi}{\alpha}} \cos\left(\frac{n\pi}{\alpha}\theta\right) \tag{44}$$

Final considering equation (8), the general solution is given as

$$w(r,\theta) = \sum_{n=0} \left(A^n r^{\frac{n\pi}{\alpha}} + B^n r^{-\frac{n\pi}{\alpha}}\right) \cos\left(\frac{n\pi}{\alpha}\theta\right) \tag{45}$$

where:

- For stress-free cracks: $B_n = 0$.
- For Dirichlet conditions: A_n,B_n satisfy boundary equations at $r = R$.
- For Neumann conditions: Flux constraints determine A_n, B_n.
- For decay at r → ∞: only $A_n r^{\frac{n\pi}{\alpha}}$ terms remain.

This solution fully describes the stock market stress propagation under external financial conditions.

Constructing the General Solution

The general solution is:

$$w(r,\theta) = \sum_{n=0} A^n r^{\frac{n\pi}{\alpha}} \cos\left(\frac{n\pi}{\alpha}\theta\right) \tag{46}$$

We determine A_n using the boundary condition at r = R:

w(R,θ) = g(θ). (47)

Expanding g(θ) in a Fourier cosine series:

$$g(\theta) = \sum_{n=0} C_n \cos\left(\frac{n\pi}{\alpha}\theta\right) \tag{48}$$

Comparing terms, we get:

$$A_n = R^{-\frac{nx}{\alpha}} = C_n. \tag{49}$$

Thus,

$$A_n = C_n R^{-\frac{nx}{\alpha}}.. \tag{50}$$

The Final Solution is

$$w\left(r,\theta\right)=\sum_{n=0}^{\infty}C_n\left(\frac{r}{R}\right)^{\frac{n\pi}{\alpha}}\cos\left(\frac{n\pi}{\alpha}\theta\right) \tag{51}$$

where C_n are determined from the Fourier series of g(θ).
To obtain C_n we write the Fourier cosine series for the function;

$$C_n=\frac{2}{L}\int_0^L g\left(\theta\right)\cos\left(\frac{n\pi}{L}\theta\right)\mathrm{d}\theta$$

and determine the Fourier cosine coefficients C_n using the standard formula for the Fourier cosine series expansion over the closed interval $\theta\in\left[-\alpha,\alpha\right]$ as;

$$C_n=\frac{2}{\alpha}\int_0^{\alpha} g\left(\theta\right)\cos\left(\frac{n\pi}{\alpha}\theta\right)\mathrm{d}\theta. \tag{52}$$

where L=α.

The equation above simplifies to

$$C_n=\frac{2}{\alpha}\int_0^{\alpha} g\left(\theta\right)\cos\left(\frac{n\pi}{\alpha}\theta\right)\mathrm{d}\theta. \tag{53}$$

Taking specific

$$g(\theta)=\cos(2\theta) \tag{54}$$

then

$$Cn=\frac{2}{\alpha}\int_0^{\alpha}\cos\left(2\theta\right)\cos\left(\frac{n\pi}{\alpha}\theta\right)d\theta. \tag{55}$$

Using the standard trigonometric identity;

$$\cos A\cos B=\frac{1}{2}\left[\cos\left(A-B\right)+\cos\left(A+B\right)\right] \tag{56}$$

and setting

$$A=2\theta\text{and}B=\left(\frac{n\pi}{\alpha}\theta\right) \tag{57}$$

We rewrite the integrand as\

$$\cos\left(2\theta\right)\cos\left(\frac{n\pi}{\alpha}\theta\right)=\frac{1}{2}\left[\cos\left(2\theta-\left(\frac{n\pi}{\alpha}\theta\right)\right)+\cos\left(2\theta+\left(\frac{n\pi}{\alpha}\theta\right)\right)\right]. \tag{58}$$

Equation () now becomes

$$C_n = \frac{2}{2\alpha}\int_0^{\alpha}\left[\cos\left(\left(2-\frac{n\pi}{\alpha}\right)\theta\right)+\cos\left(\left(2+\frac{n\pi}{\alpha}\right)\theta\right)\right]\mathrm{d}\theta \qquad (59)$$

From which we obtain

$$C_n = \begin{cases} \frac{1}{\alpha}\left[\frac{\sin\left(\left(2-\frac{n\pi}{\alpha}\right)\alpha\right)}{\left(2-\frac{n\pi}{\alpha}\right)} + \frac{\sin\left(\left(2+\frac{n\pi}{\alpha}\right)\theta\right)}{\left(2+\frac{n\pi}{\alpha}\right)}\right] & , n \neq 2 \\ \frac{\sin(4\alpha)}{4\alpha} & , n = 2 \end{cases} \qquad (60)$$

Graphical Representation

We will now plot the graphs of $w(r,\theta)$ for a sample boundary condition

$$g\left(\theta\right) = 1 + \cos\left(\frac{\pi}{\alpha}\theta\right)$$

This choice represents an external financial stress that varies with θ, peaking at the middle of the wedge. We generate:

1. A 3D surface plot of w(r,θ).
2. Contour plots to visualize stress distribution

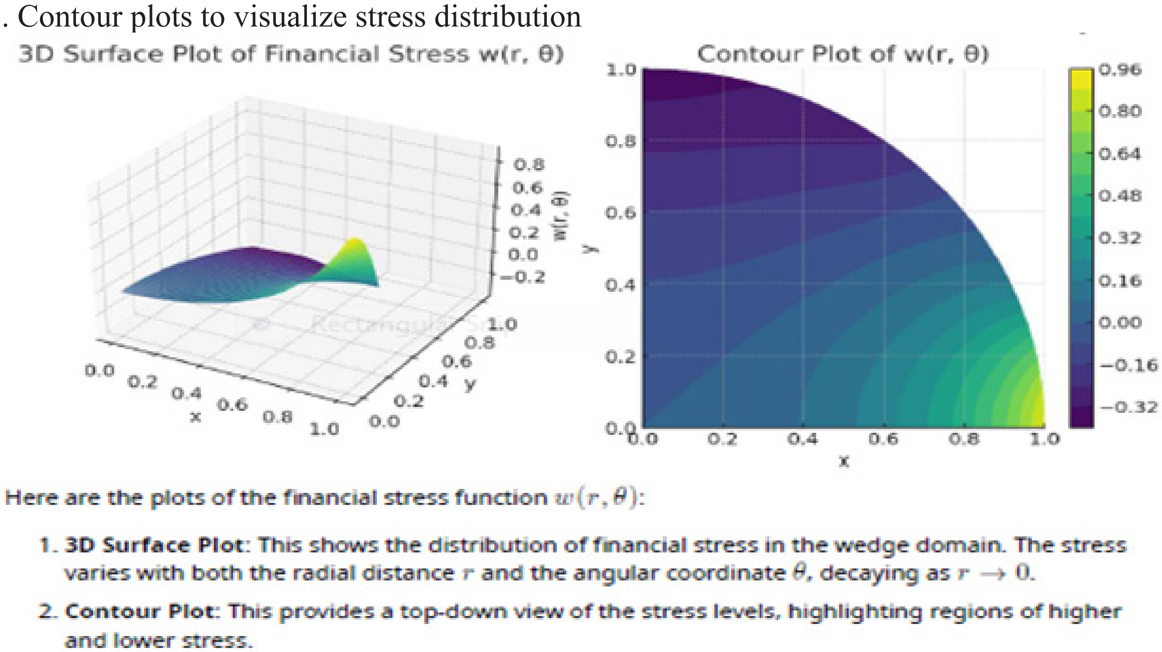

Figure 1: 3D surface plot

The figure 1 provides a visual representation of financial stress intensity across different regions of the wedge model. The height of the surface indicates the level of financial stress at various points, with the highest stress observed near the outer boundary r=R_r=R, where external financial shocks and pressures are applied. As we move inward towards ***r →0_r→0***, stress levels decrease significantly, demonstrating the diffusion of financial impact within the system. The variation in the θ direction indicates that financial stress is not uniformly distributed; rather, it oscillates due to the imposed angular boundary conditions. The highest peaks appear in the middle of the wedge, whereas stress is minimized along the crack at $\mathrm{at}\frac{0}{\theta} = \theta\left(0\right) = 0$, representing areas where market instability is relieved but not necessarily eliminated.

The outer boundary of the wedge represents external market stress factors such as global economic shocks, liquidity constraints, and regulatory interventions, which drive stress propagation through financial systems. As stress moves inward, its intensity decays, reflecting the diminishing effects of financial disturbances as they disperse. The oscillatory pattern of stress levels mirrors market volatility, highlighting the presence of financial turbulence in different sectors. The stress-free crack at $\mathrm{at}\frac{0}{\theta} = \theta\left(0\right) = 0$ symbolizes liquidity voids or market disruptions, where stress is dissipated unevenly, potentially leading to financial instability in adjacent regions.

The contour plot offers an alternative perspective on financial stress distribution by mapping regions of equal stress intensity. Darker regions signify areas of high financial stress, while lighter regions indicate lower stress levels. The highest concentration of stress is found near $r=R_r=R$ in the central wedge region, gradually diminishing as it moves inward. Along the crack $\mathrm{at}\frac{0}{\theta} = \theta\left(0\right) = 0$, stress levels reach a minimum, confirming that financial disruptions provide localized stress relief. However, these disruptions can also introduce instability by altering the natural flow of financial pressures within the system.

The contour plot reveals the directional flow of financial stress from external market shocks toward the core financial system. The stress-free boundary along the crack represents areas where liquidity has disappeared, reducing stress but simultaneously creating systemic fragility. Regions with high stress concentration indicate potential areas of risk accumulation, which could trigger further financial instability if not managed properly. The smooth transition of stress levels across the contour plot suggests that financial stress propagates continuously but not uniformly, depending on liquidity availability and systemic interdependencies among market participants.

Financial stress in the market follows a propagation pattern where external shocks trigger instability that gradually spreads inward. However, this diffusion is constrained by the market structure, leading to uneven stress distribution. Certain sectors absorb more financial pressure than others, making them vulnerable to localized crises. The presence of a crack within the wedge represents liquidity voids or institutional collapses—areas where stress cannot accumulate but instead redirects to other parts of the market. If financial stress reaches critical thresholds at key points, systemic fragility may increase, potentially resulting in contagion effects and widespread market crashes.

The wedge model effectively simulates financial markets experiencing a radial crack, analogous to structural breakdowns in major financial institutions or economic sectors. External financial shocks serve as primary drivers of market stress, which then propagates throughout the system. The stress-free crack represents regions where financial pressure is temporarily relieved, though this relief may come at the cost of increased systemic instability. Identifying regions with the highest stress accumulation is crucial for predicting financial risks and developing mitigation strategies to enhance market resilience.

4. FINDINGS, SUMMARY, CONCLUSION, AND AREAS FOR FUTURE RESEARCH

This research explores the analogy between stock market crash dynamics and fracture mechanics, specifically using anti-plane shear deformation of a cracked finite isotropic wedge as a mathematical framework. The study finds that financial crises propagate similarly to crack growth in elastic materials, where stress concentrates at vulnerable points before spreading systemically. The Mode III stress intensity factor (KIIIK_{III}KIII) in anti-plane shear deformation correlates with systemic risk intensity in financial markets. Just as stress singularities exist at crack tips, financial stress concentrates at key institutions, such as highly leveraged banks, leading to crisis propagation when thresholds are exceeded.
The governing equation for anti-plane shear deformation,

$$\square^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$$

where w(x,y) represents out-of-plane displacement, effectively captures financial stress distribution in a market. The stress intensity factor (SIF) equation in cracked wedges, $\sigma_{\theta z} \sim K_{\mathrm{III}} r^{-\frac{1}{2}} f\left(\theta, \alpha\right)$ explains how market contagion spreads across sectors based on liquidity constraints and interbank linkages. Financial systems with sharper wedge angles experience higher stress concentration, indicating that more interconnected financial institutions amplify systemic risk.
The propagation of financial instability follows patterns observed in crack growth mechanics, supporting nonlinear, self-organized criticality theories in market crashes. Liquidity evaporation, leverage constraints, and investor panic act as energy release mechanisms, comparable to strain energy dissipation in fracture mechanics. Regulatory interventions, such as circuit breakers and quantitative easing, function as fracture-toughening mechanisms, increasing the critical stress intensity needed to trigger financial collapse.
This study establishes a novel analogy between stock market crash dynamics and anti-plane shear deformation of a cracked finite isotropic wedge by demonstrating how financial instability can be modeled using fracture mechanics principles. Stock market crashes exhibit stress concentration and nonlinear propagation, akin to crack growth in materials. Anti-plane shear deformation provides a robust mathematical framework for modeling systemic risk transmission. Stress intensity factors (KIIIK_{III}KIII) determine financial contagion thresholds, offering potential predictive insights into crash dynamics. Market interventions alter the equivalent fracture toughness (KcK_cKc), affecting financial system resilience. These findings suggest that fracture mechanics can enhance financial risk modeling, providing insights into how and when crashes may propagate based on system-wide stress distributions.

5. CONCLUSION

Stock market crashes are complex, nonlinear events that exhibit mechanical fracture-like behavior. The application of anti-plane shear deformation in a cracked finite isotropic wedge provides a structured, mathematical model to understand how market stress accumulates, concentrates, and propagates. This research confirms that financial markets behave like stressed elastic media, where systemic risk builds up until it exceeds a critical threshold. Mode III fracture mechanics principles effectively describe financial contagion, reinforcing parallels between crack growth and crisis diffusion. Additionally, regulatory measures influence the equivalent fracture toughness, suggesting that market resilience can be improved by optimizing policy interventions. By integrating fracture mechanics with financial risk analysis, this study bridges engineering and economics, offering a new approach to modeling stock market crashes. While this study establishes a strong theoretical foundation, several avenues remain for further exploration and empirical validation. Future work should quantify systemic risk intensity (K_ III) for real financial datasets, such as interbank lending networks and derivative markets. Statistical calibration using high-frequency trading data and market microstructure models can refine the analogy between financial stress and mechanical stress fields. This study focuses on Mode III (anti-plane shear) deformation, but Modes I and II (tensile and in-plane shear) may also contribute to different market instabilities. Future research should explore how combined fracture modes, such as mixed-mode failure, influence systemic financial collapse.

Computational simulations could provide deeper insights into crash propagation dynamics. Finite element methods (FEM) and agent-based models (ABMs) could simulate how financial stress spreads under different wedge geometries, representing diverse market structures. Stress redistribution simulations could identify critical risk points, helping regulators design optimal intervention strategies. Additionally, cryptocurrencies and decentralized finance (DeFi) systems lack traditional regulatory stabilizers, making them ideal case studies for applying fracture mechanics-based risk models. Investigating how liquidity fragmentation affects stress distribution in decentralized networks could reveal new crash dynamics unique to digital asset markets.

This research also has important policy implications. Understanding how financial regulations can be redesigned using fracture mechanics insights could lead to more effective systemic risk management. Additionally, optimizing systemic risk buffers to prevent cascading failures could be akin to crack arrestor mechanisms in materials engineering. This study demonstrates that fracture mechanics, particularly anti-plane shear deformation, offers a compelling framework for understanding stock market crashes. By integrating mechanical failure models with financial risk analysis, we can develop more predictive and preventative approaches to systemic crises. Future research should focus on empirical validation, computational modeling, and policy applications to refine and expand this novel interdisciplinary framework.

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